Lester Lees and Hypersonic Aerodynamics

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Six decades after the Wright Brothers' first airplane flight, technology had advanced to a level that made hypersonic flight achievable. This incredible feat required major developments in aerodynamics, propulsion, structures, materials, and control systems. One of the pioneers in the development of hypersonic aerodynamics was Lester Lees, a noted teacher and researcher at Princeton University and the California Institute of Technology. Lees advanced the knowledge base in hypersonic aerodynamics with work on boundary layers and heat transfer, flow over ramps, cones, and blunt bodies, and the development of blunt-nosed reentry vehicles. He also made great progress toward the maturation of the kinetic theory of gases. The advances made by Lees and his students enabled, to a great degree, the development of hypersonic flight programs and the U.S. space program. To commemorate these events, this paper presents a brief biographical sketch of Lester Lees including some anecdotes about his life, presents his work in various fields of engineering, and discusses his contributions to hypersonic aerodynamics.

Nomenclature

a = acoustic speed

 C_p = pressure coefficient, $\equiv (p - p_{\infty})/q_{\infty}$ $C_{p,\text{max}}$ = stagnation pressure coefficient at M_{∞}

d = characteristic body diameter Kn = Knudsen number, $\equiv \lambda/\ell \propto M_{\infty}/Re$

 ℓ = characteristic body length M = Mach number, $\equiv U/a$

p = pressure

 p_{0_2} = stagnation pressure behind a normal shock

q = dynamic pressure, $\equiv (\gamma/2)pM^2$ Re = Reynolds number, $\equiv U_{\infty}\ell/\nu$ U = fluid velocity

 γ = ratio of specific heats θ = surface inclination angle

 κ = hypersonic similarity parameter, $\equiv M_{\infty} \tau$

 λ = mean free path of gas particles μ = Mach angle, $\sin^{-1}(1/M)$ ν = kinematic viscosity τ = body slenderness ratio, d/ℓ ∞ = freestream condition

Introduction

N 17 December 1903, Wilbur and Orville Wright made the first sustained, heavier-than-air flight at Kitty Hawk, North Carolina. Whereas others had failed, the Wright's succeeded because of their scientific approach: they used theoretical analysis, wind-tunnel testing, and glider flights to verify their concepts. Their airplane reached a speed of about 35 mph and soared to an altitude of approximately 10 ft, corresponding to a Mach number of 0.046. On 22 September 1963, Robert M. White flew the North American X-15 at 4520 mph at an altitude of 354,200 ft—a Mach number

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of 6.7! This remarkable achievement was accomplished over such a short period of time as a result of intensiveresearch and development by a large number of scientists and engineers. One of the pioneers among the group working on hypersonic aerodynamics was Lester Lees (1920–1986). To commemorate the centennial of human flight, we present a paper about Lester Lees, including a brief biographic sketch, personal anecdotes, and a discussion of his contributions to aerodynamics, particularly to hypersonic aerodynamics.

In 1946, H. S. Tsien wrote the first paper on hypersonic aerodynamics, coining the word "hypersonic" to describe the unique features of flow in the "ultra" supersonic flight regime. Tsien's paper followed the work of von Kármán, who wrote a paper on the similarity law of transonic flow (which was published after Tsien's paper). Because Tsien's work followed that of von Kármán, the velocity potential formulation was employed as the theoretical framework for devising the similarity laws for hypersonic flow. The theoretical hypersonic concepts were generalized to rotational flow behind curved strong shock waves by Hayes the next year. (In fact, the hypersonic similarity law can also be derived directly from Euler's equation. (4)

To extend the work to viscous hypersonic flow in the late 1940s, H. S. Tsien suggested that S. F. Shen at the Massachusetts Institute of Technology (MIT) conduct research on the hypersonic boundary layer over a flat plate. In the early 1950s, Ting-Yi Li and Henry T. Nagamatsu of the Hypersonic Group at the California Institute of Technology (CalTech) extended Shen's work. 5 At Princeton University, Ronald Probstein, writing his Ph.D. dissertation under Lester Lees, took a more rigorous approach by obtaining similarity solutions for a hypersonic boundary layer, outside of which lies a shock wave.^{6,7} They found that the leading-edge region of the flat plate had a strong interaction with the shock wave, whereas the downstream region had a weak interaction, which also prevailed in a supersonic boundary layer. All of these developments were crucial to enabling the flight of hypersonic vehicles. Lees and his students, however, were not done making advances that would be crucial to the development of hypersonic flight.

In 1952, Lees went to CalTech on a leave from Princeton; Ronald Probstein followed him later. H. T. Yang (the senior author of this paper) was assigned to help them complete a Princeton report on hypersonics. Yang was one of Lees' first students in the Hypersonic Group at CalTech, which was under the renewed leadership of Clark B. Millikan and Lester Lees. It is interesting to note, therefore, that this paper is being written by Lester Lees' academic "son" and "grandson" because H. T. Yang was one of Lester Lees' Ph.D. students at CalTech and R. M. Cummings was one of H. T. Yang's Ph.D. students at the University of Southern California.

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Lees continued to work on hypersonic research while at CalTech, directing many graduate students on a variety of research endeavors. When taken as a whole, his developments in hypersonic flow theory were crucial to the development of hypersonic vehicles (such as the X-15) and the space program (including the Mercury, Gemini, and Apollo programs). His developments included advances in inviscid flow theory (especially the development of modified Newtonian theory), viscous flow theory (including the hypersonic boundary-layer equations), high-temperaturegasdynamics, and the kinetic theory of rarefied gases and stand as essential elements in our understanding of hypersonic flow. Without these developments the major advances in manned space flight of the 1960s would not have been possible. We dedicate this paper to the memory of Lester Lees, a pioneer in hypersonic aerodynamics, a gifted scientist, and a devoted mentor to many students.

Lester Lees: Professor, Engineer, Scientist

Brief Biographical Sketch

A memorial tribute to Lester Lees was written in 1992 by F. E. Marble. Interested readers are referred to this enjoyable paper for further details about Lester Lees' life and work. We will only give a brief biographical sketch in this paper, but we will complement it with some anecdotes in the next subsection. Lester Lees was born in New York City on 8 November 1920, and passed away on 10 November 1986, two days after his 66th birthday. He finished high school in 1937 and was admitted to MIT at the age of 16, where he received his B.S. and M.S. degrees in aeronautical engineering in 1941 (Ref. 8). Soon thereafter he joined the U.S. Air Force Air Material Command at Wright Field in Ohio, where Theodore von Kármán met him and convinced him to come to CalTech in 1942. During World War II, he was assigned to the National Advisory Committee for Aeronautics (NACA) Langley Memorial Aeronautical Laboratory, where he worked on many of the theoretical developments that would make him a well-known name in aeronautics, including his concepts on stability theory for compressible laminar flows. In the late 1940s he joined the aeronautical engineering faculty of Princeton University where he was an active participant at the Gas Dynamics Lab on the James Forrestal campus. (Information on the Princeton aeronautical engineering program is available online at http://www.princeton.edu/~mae/SHL/pic/glassman.html.) He formally returned to CalTech in 1953 and worked for the next 33 years in the Aeronautics Department and later in the Environmental Quality Laboratory (Fig. 1).

Personal Anecdotes

There are a great number of legendary stories about Theodore von Kármán; most aerodynamicistsknow some of these. Although stories about Lees are not quite as flamboyant as those of Theodore von Kármán, they are recalled with special fondness and warmth by those who worked with him. Most of the following observations and anecdotes are from the senior author of this paper, and others can be found in Ref. 8.

One of the fondest memories of Lester Lees' former students is of the "legendary" parties that he would hold in his home, co-hosted by his wife Connie. Lees was a noted gourmet and wine aficionado, while Connie was an excellent cook. They invited students, among others, to their home at Altadena for hearty meals and thought-provoking conversation. "These students were an integral part of Lester's and Connie's life; essentially they were his extended family." Figure 2 shows a birthday tribute to Lester Lees at The Huntington Hotel in Pasadena around 1980 by his students, many of whom enjoyed evenings at the Lees' home. This picture was arranged by, among others, D. R. S. Ko, who donated and dedicated on 8 January 2001 the Lees–Kubota Lecture Hall in the Guggenheim building on the CalTech campus.

Lester Lees was also quite well known for enjoying a good argument. When Lees discussed topics with his colleagues or students, he would address them by their name if he agreed with them. Otherwise, if he did not agree, he would call the person "my friend," which ensured that a good argument was soon to follow.

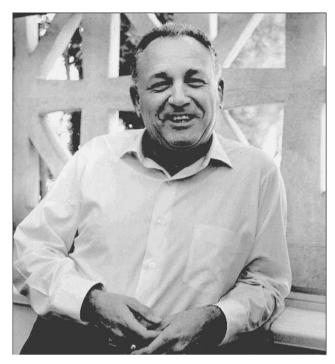


Fig. 1 Lester Lees at CalTech (courtesy of the Archives, California Institute of Technology, Pasadena, California).

As has been true with other well-known aerodynamicists (R. T. Jones for example), Lees did not pursue his doctorate; he was, however, an outstanding scholar. In a 2 February 1947 Langley group photo hung on his office wall, with Dr. T. von Kármán, Dr. H. L. Dryden, Dr. H.S. Tsien, and 26 others, most are labeled as "Dr.," including Lees; he dutifully corrected the "D" on the photo to "M(r. Lees)." When H. T. Yang was on doctoral committees, some colleagues would say that all members should posses a doctoral degree. Remembering his mentor, Lester Lees, Yang would remark that if this were the case there would never have been a first doctorate degree.

Although Lees was a prolific researcher (see the Appendix for a list of his publications), he did seem to take his work modestly. After completing some research results, he was known to say, "What we are doing is not going to shake the foundation of the Earth." In truth, however, much of his work laid the very foundations for the development of hypersonic flight and space travel.

In addition to his technical mentoring of students, Lees also taught about the various professional aspects of being an engineer. He told them that to write a good paper they should assume the reader does not know anything about the subject matter. Lees would say, "Like a symphony, in which the theme is repeated many times, so are the main points of the paper repeated as necessary." He also was ahead of his time in asking that his students not use the passive voice in their writing. He said to avoid using, "It is well known...," or "It can be easily shown that...."

Another aspect of professionalism displayed by Lees was his refusal to publish the same work over and over again. In the "publish or perish" days of the time, some authors presented the same paper or its variation several times. For example, in the late 1950s the annual meeting of the Institute of the Aeronautical Sciences was held at the Statler Hotel in midtown Manhattan Monday through Thursday. The Fluid Dynamics Division of the American Physical Society held its meeting at the Hotel New Yorker in lower Manhattan Friday and Saturday. The latter conference often featured repeat performances of papers from the former conference, with the same audience and same presenters showing the same results. When frequently invited to speak at such events, Lees would often decline by saying, "I have nothing new to report at this time," to avoid repetition merely for the sake of padding his curriculum vitae.

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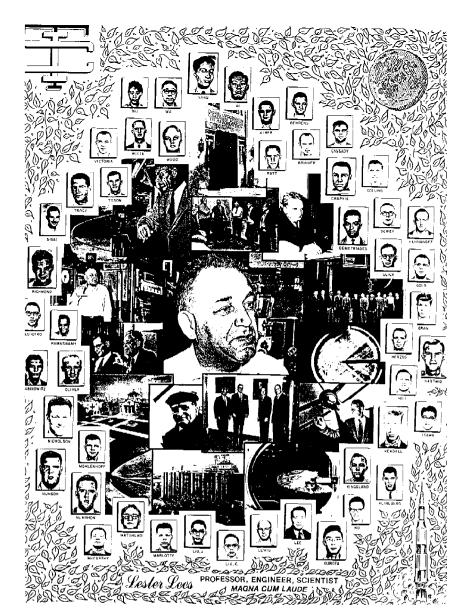


Fig. 2 Lester Lees CalTech Tribute: Professor, Engineer, Scientist; Magna Cum Laude (includes pictures of his CalTech students: Ai, Alber, Batt, Behrens, Brinker, Cassady, Chapkins, Collins, Demetriades, Dewey, Fernandez, Glick, Gold, Gran, Herzog, Hartwing, Hill, Igawa, Kendall, Kingsland, Klineberg, Ko, Kubota, Lee, Lewis, C. Liu, J. Liu, Marlotte, Matthews, McCarthy, McMahon, Munson, Mohlenhoff, Nicholson, Oliver, Rabinowitz, Ramaswamy, Reshotko, Richmond, Sigal, Tracy, Tyson, Voctoria, Witte, Wood, Wu, Wu, and Yang).

Lees not only worked in a variety of scientific and engineering areas, including solar wind and the environment, but he was also interested in cultural events and community affairs. In addition to the various committees he served on as mentioned in Ref. 8, he succeeded Clark B. Millikan as the Chairman of the CalTech Faculty Committee on the Jet Propulsion Laboratory. Furthermore, he even ran one time for public office in Altadena by simply listing himself as, "Lester Lees, Teacher." His interests and pursuits of knowledge seemed to have no bounds.

Lees was also very supportive of, and engaged in, the future careers of his students, which sometimes meant that he helped them gain employment, but at other times meant that he would discourage students from taking jobs that he was not convinced would enhance a student's future. After H. T. Yang completed his Ph.D in 1955, he was retained as a Research Fellow in the Hypersonic Group at CalTech. [That year CalTech graduated 8 Ph.D.s in aeronautics, making a total of 115 since 1932 (Ref. 10).] One day a notice appeared on the bulletin board announcing a job opening at the Institute for Fluid Dynamics and Applied Mathematics at the University of Maryland. Yang requested some information and received an invitation from his former teacher Shih-I Pai (a 1940 Ph.D. student under von Kármán) to join the group headed by himself and

J. M. Burgers (a contemporary of, and coauthor with, von Kármán). When the senior author left CalTech for Maryland, Lees told him, "My friend, you will be sorry." Apparently in this situation Lees' opinion turned out to be somewhat ironic—if the senior author had not left CalTech, he would not have met the first author, and this paper would not have been written. But more frequently Lees' advice and aid was crucial to the success of his students. All in all, Lees' students have made significant contributions to aerodynamics, in part due to his mentoring.

Work in Fields Other Than Hypersonic Aerodynamics

Early in his career, Lees worked on a variety of aerodynamic research topics prior to begining his work on hypersonics. Primary among these topics were compressibility corrections and boundary-layer stability. In addition, later in his career he became interested in environmental issues and made important contributions to that field as well. This section describes some of Lees' contributions in areas of research other than hypersonics.

The Prandlt–Glauert compressibility rule was successfully applied to two-dimensional subsonic flow in the 1920s. 11 The compressibility correction allowed experimental data or results from

incompressible potential flow theory to be extended to various subsonic flight conditions by multiplying velocities and pressures by the factor $1/\sqrt{(1-M_\infty^2)}$. The extension of the compressibility correction to three-dimensional flow was not straightforward; the difficulty, as first pointed out by Göthert in 1940, was caused by the boundary conditions. Lees did a thorough analysis in 1946 (Ref. 13) and corrected his previous work with Tsien on the subject by stating that the velocity and pressure variation over a slender body was given by $1/(1-M_\infty^2)^{3/2}$, a significant difference from the Prandtl–Glauert correction. The discovery of the new form of the factor allowed for a more appropriate compressibility correction for fuselages and missile bodies, an important contribution at the time.

The stability of laminar boundary layers was studied by C. C. Lin, among others, in a series of papers during the early 1940s. ^{15–18} In collaboration with Lin, Lees investigated the stability of laminar boundary layers in compressible flow, ¹⁹ and Lees later continued the work on his own. ^{20,21} The results of this ground-breaking work enabled others (such as van Driest²² and Spalding and Chi²³) to create practical engineering methods for the prediction of skin friction and heat transfer at supersonic speeds, which aided in the development of airplane, missile, and launch vehicle technology.

Early in the 1970s CalTech created the new (and timely) Environmental Quality Laboratory. The President of CalTech at the time, Harold Brown, asked Lees to become the first director of the lab. "Under his stimulating guidance, this became a flourishing and prolific organization." Lees did not shy away from making public comments about the poor state of the environment in Southern California, including "before their time" opinions about smog, power plants, and the impact of population growth on environmental quality (see Ref. 24 for example). The amazing aspect of his work at the Environmental Quality Laboratory is that he continued developing concepts in hypersonic aerodynamics and returned to his position as Professor of Aeronautics in 1974.

Lees' Contributions to Hypersonic Aerodynamics

The hypersonic flow regime is considered to be a distinct flow regime from supersonic flow because a variety of distinguishing effects takes place when the freestream Mach number reaches approximately $M_{\infty} \approx 5$. Although the hypersonic flow regime has no precise Mach-number range and the preceding value serves only as a rule of thumb, hypersonic flow begins when the flow experiences some of the following five distinguishing effects²⁵: 1) thin shock layers, 2) entropy layers caused by curved shocks, 3) viscous-inviscid interactions, 4) high-temperature effects (such as dissociation) and extreme heat transfer, and 5) low-density flows such as rarefication (typically due to reentry or attempts to achieve orbit or high altitude).

The extreme nature of the hypersonic flow regime can have profound effects on the design of flight vehicles (as shown in Fig. 3). The high temperatures and pressures associated with hypersonic flight force designers to create lifting-body shapes with blended wing surfaces. The flight conditions just listed (usually taking place

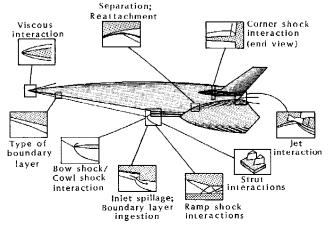


Fig. 3 Characteristics of hypersonic flow (from Ref. 25).

at high altitudes) also make it difficult to model hypersonic flight accurately in wind tunnels because of the inability of tunnels to match the Mach and Reynolds number (as well as high enthalpy) characteristics of atmospheric flight. These technical challenges make the accomplishments of Lester Lees even more important and impressive. Lees developed theoretical models that enabled designers to create hypersonic vehicles in spite of the complexity of the flow-field, an accomplishment that is all the more amazing considering that his theories and concepts are still widely used to this day. For more detailed reviews of hypersonic flow theory and research, see Refs. 4 and 25–27. Lees conducted a logical and ever-widening set of research into hypersonic aerodynamics, beginning with inviscid flow theory, eventually adding viscous effects, and finally including high-temperatureand rarefied gas effects in his models. Each of these developments will be addressed individually.

Inviscid Flow Theory

Lees began his work in hypersonic flow by trying to relate hypersonic flow to supersonic flow, following the well-established similarity concepts of the time. Because compressibility effects had first been taken into account by similarity to incompressible flow, most researchers continued to work on this notion for supersonic flow and eventually hypersonic flow. Tsien had developed a hypersonic similarity parameter $\kappa = M_{\infty}\tau$, where τ is the slenderness ratio of the body (which is the inverse of the body fineness ratio ℓ/d), and he showed that the parameter could be used to relate flows at different Mach numbers and body shapes.¹ In other words, two bodies in hypersonic flow at different Mach numbers will display identical aerodynamic characteristics if the product of the Mach number and slenderness ratio is identical. Lees "put legs" on this theory by analytically discovering the limitations of applicability in both Mach number and body slenderness ratio. Lees found that as long as the body was relatively thin (the cone half-angle was less than approximately 24 deg) then the hypersonic similarity parameter was valid throughout the hypersonic flow region, as well as most of the supersonic flow region.²⁸ This meant that already existing supersonic results for cones²⁹ could be used for hypersonic flow as well, which was an important conclusion. Lees also hypothesized that these results would work for cones at an angle of attack and for ogival bodies—more results that showed great applicability.

After solving the "building block" flow for cones, Lees continued by investigating the applicability of similarity rules for hypersonic flow over ramps (or wedges).³⁰ Lees' work on ramps led to conclusions similar to the previous cone work, which created a method for treating two- and quasi-two-dimensional flows over slender bodies because both hypersonic flow over wedges and cones could be related to their supersonic counterparts. With this work Lees was quickly helping to develop methods that would allow for the inviscid analysis of hypersonic vehicles at an engineering application level.

Perhaps the best known of Lees' theoretical developments for hypersonic flow was his extension to Newtonian flow theory. Newton had postulated the forces acting on solid surfaces based on the assumption that the component of momentum normal to the surface was lost during the interaction of the fluid with the surface.³¹ This led to a rather simple result for the pressure coefficient acting on a surface inclined at an angle θ to the freesteam flow:

$$C_p = 2\sin^2\theta \tag{1}$$

Newton's flow theory did not do a good job predicting the pressures in incompressible flow and was unduly blamed to have delayed human flight because of the low lift-to-drag ratio predicted, but his basic assumptions matched hypersonic flow quite well. ³² As the freestream Mach number increases, the Mach angle $\mu = \sin^{-1}(1/M_{\infty})$ approaches zero. As the Mach angle continues to decrease, the assumptions of Newton's theory become approximately satisfied because the shock decreases the normal component of velocity, analogous to the flow losing its normal component of velocity. Therefore, Newtonian flow theory is technically valid for a freestream Mach number of $M_{\infty} \to \infty$.

Lees noted that Newtonian flow theory yielded a stagnation pressure coefficient of $C_p = 2$ when $\theta = 90$ deg. However, when

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reviewing the available experimental data for hypersonic stagnation pressures he noticed that the stagnation pressure coefficient never reached a value of $C_p = 2$. Based on these observations, Lees proposed that the Newtonian flow theory should be modified for noninfinite Mach numbers as³³

$$C_p = C_{p \max} \sin^2 \theta \tag{2}$$

where $C_{p \text{ max}}$ could be supplied from experimental data or by using

$$C_{p \max} = \left(p_{0_2} - p_{\infty}\right) / q_{\infty} \tag{3}$$

where p_{0_2} is the stagnation pressure behind a normal shock at the freestream Mach number (obtained from the Rayleigh pitot formula). This approach has become known as modified Newtonian flow theory and supplies engineers with a straightforward way to find pressures on surfaces in spite of the complexities of the hypersonic flowfield. An immediate, and important, application of these concepts was for reentry vehicles, which for the purposes of relieving heat-transfer problems use blunt shapes for the nose. Modified Newtonian theory allowed for the analysis of pressures on bluntnosed vehicles and became an important tool in their development—most previous surface inclination methods had been limited to small angles and could not have been properly used for blunt noses.

Viscous Flow Theory

The research work of Lees for hypersonic boundary layers and heat transfer was a natural extension of his previous work for supersonic viscous flow, but followed a more difficult path.²⁰ H. S. Tsien, while still at MIT, suggested that S. F. Shen conduct part of his doctoral thesis on the hypersonic boundary layer over a flat plate.³⁴ The approach uses the Karman-Pohlhausen integral method by assuming a linear velocity profile across the boundary layer, which is supposed to coincide with the shock wave. In the early 1950s, Ting-Yi Li and Henry T. Nagamatsu of the Hypersonic Research Group at CalTech extended Shen's work by assuming a quartic velocity profile.⁵ At Princeton, Ronald Probstein, writing his Ph.D. dissertation under Lester Lees, took a more rigorous approach by obtaining similarity solutions for a hypersonic boundary layer, outside of which lies a shock wave. ^{6,7,35} They found that the leading-edge region of the flat plate had a strong interaction with the shock wave, whereas the downstream region had a weak interaction, which also prevailed in a supersonic boundary layer.

In his 1955 survey paper³³ Lees mentioned the joint effort with Li and Nagamatsu to clarify the two different approaches. When the momentum integral method employed by Li and Nagamatsu is carried out in Howarth coordinates for density transformation, the results obtained by Stewartson agree within 2% with those obtained by Lees and Probstein. However, the Li and Nagamatsu method is conceptually incorrect by assuming the shock wave and boundary layer coincide, which violates the conservation of mass. Furthermore, the Li and Nagamatsu approach was limited only to strong interactions.

At the CalTech Hypersonic Research Group, Lees continued this work and directed, with Clark B. Millikan, theoretical and experimental research in hypersonic flow. He also consulted on the topic, mainly for Ramo and Wooldridge, who left Hughes Aircraft Company to start the Intercontinental Ballistic Missile (ICBM) program in the 1950s. Some of Lees' publications on hypersonic flow are a direct result of this effort.

In his 1957 survey paper³⁶ the effect of the blunt tip of the flat plate on the theoretical infinitely thin leading edge was addressed; Lees found blast-wave theory useful in this case. In obtaining the similarity solution for the hypersonic boundary layer, the transformation of the streamwise and normal independent variables was named on p. 229 of Ref. 4 as the Lees–Dorodnitsyn transformation. Lees also continued his stability work with Reshotko and others (see list of publications in the Appendix) and quickly followed up with the development of the hypersonic boundary-layer equations³⁷ and applications to flow over ramps³⁰ and blunt-nosedbodies (including stagnation-region heating).³⁸

High-Temperature Gas Dynamics

High-temperature effects occur in hypersonic external flow over vehicles such as spacecraft and in internal flows in powerplants such as rockets. In the case of atmospheric flight, the kinetic energy of high-speed air around the vehicle is converted into thermal energy, which is aerodynamic heating. The vibrational degree of air molecules (it is sometimes expedient to treat air as a simple gas) will be excited at about 800 K. At 1 atm the oxygen molecule starts to dissociate at about 2000 K, and nitrogen starts at 4000 K; above 9000 K ionization will take place. Therefore chemical reactions and electromagnetic effects will take place within the flowfield.

Lees, following the suggestion of S. S. Penner, treated air as a simple gas and considered only binary diffusion in his laminar heattransfer work.³⁸ The chemical processes are very complicated, and not all of the rate constants are known, and so he considered two limiting cases. One is equilibrium flow where the chemical process is very fast and thermodynamic equilibrium is established instantaneously. The other is frozen flow where the process is very slow, the chemical process does not take place, and the air molecules remain dissociated. The solid boundary is assumed catalytic or not, with and without mass addition. The pressure is also needed to analyze the heat transfer. It turned out his modified Newtonian formula worked very well. The underlying reason is that pressure is a mechanical entity, not affected by the chemistry. The real-life situation lies within the fortunately not too large ballpark bounded by those limiting cases. Lees was not content to stop there, extending his work to also study the ablation phenomenon, which would have crucial importance to hypersonic vehicles such as the X-15. A good summary of his results for ablation can be found in Ref. 39.

Kinetic Theory of Rarefied Gases

As the speed and altitude of a flight vehicle increases, the Mach number also increases while the Reynolds number decreases. As a result, the Knudsen number Kn (which can be expressed as the ratio of Mach number to Reynolds number) increases. The Knudsen number is the ratio of the mean free path of the atmospheric gas particles relative to the characteristic length of the vehicle. Therefore for Knudsen numbers greater than 0.1, the continuum gas dynamics assumption is no longer valid, and the kinetic theory of rarefied gas dynamics should be employed.

The first article on the mechanics of rarefied gases was written by Zahm in 1934 (Ref. 40) which was only of academic interest at the time. In 1946, Tsien⁴¹ wrote an important paper on this branch of fluid mechanics as it became practical as a result of the progress being made in propulsion. Tsien directed the first Ph.D. dissertation in this area by Richard Schamberg in 1947 at CalTech, and then Tsien left for MIT. The experimental and theoretical research on rarefied gases was picked up at University of California, Berkeley, under Samuel A. Schaaf, Lawrence Talbot, Frederick S. Sherman, and others. In the early 1950s the senior author resumed the study of rarefied gas dynamics in the Hypersonic Group under Henry Nagamatsu and Ting-Yi Li. The first approach was using Burnett equations and Schamberg boundary conditions. At the same time the Grad 13-moment equations and boundary conditions, which contain the Burnett equations and boundary conditions, were also studied. When Lees came to CalTech, the senior author was assigned to him and the research continued using Grad's equations.

Lees' ingenuity showed up in at least two aspects while directing H. T. Yang's dissertation. First, the boundary conditions had been derived mathematically by Grad, but Lees was not totally satisfied with this approach. Lees directed Yang to derive them based on the physical conservation laws of mass, momentum, and energy at the solid boundary. The second ingenious aspect relates to the solution of the Rayleigh problem by an inversion of the Laplace transform. To solve the linearized partial differential equations, the method of Laplace transforms was employed. During that time, computers were not yet widely used, and numerical inversions of the Laplace transform had not yet been developed. To invert the transform, most researchers employed small and large time approximations, a fairly typical approach at the time. But to invert for the entire time range, Lees took a new step that was out of the ordinary. He suggested that

the transformed solution at the boundary be plotted and approximated by terms from the large and small ends of the transformed variable, corresponding to small and large time. Graphically the approximate curve is very close to the exact one. The approximate transform was readily inverted and gave excellent results for the skin friction of the Rayleigh problem, whose value transits from classical friction for large time to free molecule instead of infinity for initial time. Lees told the senior author, "I talked to C. C. Lin (on sabbatical from MIT) about this inversion. He said this method could not be proved rigorously but it worked." This shows the insight Lees had with applying mathematics to physical problems, which is similar to Heaviside's invention of operational calculus, and equivalently the Laplace transform. After completing his thesis, Yang continued to work with Lees on rarefied gas dynamics (see the publication list in the Appendix for details). It is to be noted that Rayleigh's problem as well as Couette flow could be considered as "poor man's" versions of the more difficult flat-plate problem.

Lees then directed Ph.D. theses in rarefied gas dynamics by K. Y. Ai, Y. C. Wu, and C. Y. Liu. Lees devised an integral method employing a two-sided discontinuous molecular velocity distribution function. Liu under Lees direction solved the nonlinear Couette flow and other problems. ^{43,44} Finally, Lees wrote the definitive work on rarefied gasdynamics in 1965, "Kinetic Theory Description of Rarefied Gas Flow," which is still useful to this day. ⁴⁵ Taken as a whole, these developments in hypersonic aerodynamics served engineers well throughout the 1950s and 1960s and are still being used today. Whereas many developments of that time have fallen by the wayside of aerospace progress, Lees' contributions continue to find a useful place in hypersonic research and education.

Conclusions

Some people have said that Lester Lees was the "father of hypersonics." If that is true, then H. S. Tsien was the "grandfather" and Theodore von Kármán was the "great uncle," but that debate could go on forever! Regardless of how one classifies his contributions, however, there is little doubt that Lester Lees was positioned perfectly in the late 1940s and throughout the 1950s to make fundamental contributions to hypersonic research. His developments in inviscid flow theory (especially the development of modified Newtonian theory), viscous flow theory, high-temperature gasdynamics, and the kinetic theory of rarefied gases stand as essential elements in our understanding of hypersonic flow. Without these developments the major advances in manned space flight of the 1960s would not have been possible. The incredible advances in aeronautics and astronautics of the second half of the 20th century make a perfect capstone to the development of the airplane in the first half of the century, and place Lester Lees at the forefront of aerospace research and development.

Appendix: Publications of Lester Lees in Chronological Order

Meeting papers that were later published as journal papers are not included.

Tsien, H. S., and Lees, L., "The Glauert-Prandtl Approximation for Subsonic Flows of a Compressible Fluid," *Journal of the Aeronautical Sciences*, Vol. 12, No. 2, 1945, pp. 173–187, 202.

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